Math2050A Term1 2017
Tutorial 3, Sept 28

Exercises

1. Let (x_n) be a sequence of nonnegative numbers converging to x. Show that $(\sqrt{x_n})$ converges to $\sqrt{x}$.
   Textbook [Bartle] p.68: **3.2.10 Theorem**.

2. Let (x_n) be a sequence of nonnegative numbers. Suppose $\lim\limits_{n\to\infty}(-1)^n x_n$ exists in $\mathbb{R}$. Show that $\lim\limits_{n\to\infty} x_n \in \mathbb{R}$.

3. Let (a_n) be a sequence in $\mathbb{R}$ and $L \in \mathbb{R}$. Show that (a_n) converges to L iff every subsequence (a_{n_k}) has a sub-subsequence $(a_{n_{k_j}})$ converging to L.

4. Let (a_n) be a seqeunce of positive numbers. Suppose $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = L \in \mathbb{R}$. Show that $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = L$.

The following is left for you to check:
Let (a_n) be a sequence in $\mathbb{R}$ and $L \in \mathbb{R}$.

1. L is a subsequential limit of (a_n) iff for each $\epsilon > 0$, there are infinitely many $n \in \mathbb{N}$ such that $a_n \in (L-\epsilon, L+\epsilon)$.

2. L is not a limit of (a_n) iff there is $\epsilon > 0$ and infinitely many $n \in \mathbb{N}$ such that $a_n \notin (L-\epsilon, L+\epsilon)$.

3. Suppose (x_n) is a sequence of positive numbers and $\lim\limits_{n\to\infty} \sqrt[n]{x_n} = L$. Show that
   $$\lim_{n\to\infty} x_n = \begin{cases} 0 & \text{if } L < 1 \\ \infty & \text{if } L > 1 \end{cases}$$
   Considering sequence (n) and $(\frac{1}{n})$ respectively, we see that there is no conclusion about the convergence of (x_n) when $L = 1$. Combining with **exercise 4**, one can conclude **3.2.11 Theorem** at p.69 in our textbook [Bartle].